

DMS-SIMP and Complete QB

SIMP-Module-1

1. Problems on TAUTOLOGY, CONTRADICTION, CONTINGENCY:

Examples:

- (1) Prove that, for any propositions p, q, r, the compound propositions, are tautologies.
 i) $\{(\rightarrow) \wedge (\rightarrow)\} \rightarrow \{(\rightarrow)\}$ ii) $\{\rightarrow (\rightarrow)\} \rightarrow \{(\rightarrow) \rightarrow (\rightarrow)\}$
- (2) Determine whether the following statement is tautology or not.
 $(\rightarrow (\vee)) \leftrightarrow ((\wedge \neg) \rightarrow)$

2. Problems on LOGICALLY EQUIVALENT STATEMENTS: (using truth tables)

Examples:

- (1) By constructing the truth table, show that the compound propositions $\wedge (\neg \vee)$ and $\vee (\wedge \neg)$ are not logically equivalent.
- (2) Use truth tables to verify,
 (i) $[\rightarrow (\wedge)] \Leftrightarrow (\rightarrow) \wedge (\rightarrow)$ (ii) $[(\vee) \rightarrow] \Leftrightarrow [(\rightarrow) \wedge (\rightarrow)]$
 (iii) $[(\leftrightarrow) \wedge (\leftrightarrow) \wedge (\leftrightarrow)] \Leftrightarrow [(\rightarrow) \wedge (\rightarrow) \wedge (\rightarrow)]$

3. Problems on LOGICALLY EQUIVALENT STATEMENT USING Laws of Logic:

Examples:

- (1) Define logical equivalence of two propositions. Prove the following logical equivalences without using the truth tables (using laws of logic):
 i) $\vee [\wedge (\vee)] \Leftrightarrow$ ii) $[(\neg \vee \neg) \rightarrow (\wedge \wedge)] \Leftrightarrow \wedge$
 (iii) $(\rightarrow) \wedge (\neg \wedge (\vee \neg)) \Leftrightarrow \neg[\vee]$
- (2) Show that $[(\vee) \wedge \neg(\neg \wedge (\neg \vee \neg))] \vee (\neg \wedge \neg) \vee (\neg \wedge \neg)$ is tautology using laws of logic.

4. Problems on TRUTH TABLES AND INDEPENDENTS OF ITS COMPONENTS:

Examples:

- (1) Find the possible truth values of p, q and r if (i) $\rightarrow (\vee)$ is FALSE
 (ii) $\wedge (\rightarrow)$ is TRUE
- (2) Show that $(\wedge (\rightarrow)) \rightarrow$ is independent of its components.
- (3) Let p, q be primitive statements for which the implication $\rightarrow$ is false. Determine the truth values for each of the following: (i) $\wedge$ (ii) $\neg \vee$ (iii) $\rightarrow$
- (4) Let p, q, r be propositions having truth values 0, 0 and 1 respectively. Find the truth values of the following compound propositions: i) $(\wedge) \rightarrow$ ii) $\rightarrow (\wedge)$
 iii) $\wedge (\rightarrow)$ iv) $\rightarrow (\rightarrow \neg)$.
- (5) Show that the truth values of the following statements are independent of their components: i) $[\wedge (\rightarrow)] \rightarrow$ ii) $(\rightarrow) \leftrightarrow [\neg \vee]$

SIMP-Module-2

(I) Problems on MATHEMATICS INDUCTION:

Examples:

(1) By the principle of .Mathematical Induction, prove that,

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(n+2)}{6}$$

(2) Define Principle of Mathematical Induction. For the Fibonacci sequence

2.Problems on BINOMIAL THEOREM:

Examples: (1) Find the coefficient of $x^9 y^3$ in the expansion of $(2x-3y)^{12}$

(2) Find the coefficient of $x^9 y^3$ in the expansion of $(2x-3y)^{12}$ etc.,

3. Problems on PERMUTATIONS, COMBINATIONS AND PRODUCT RULE:

Examples:

1) In the word ENGINEERING, (i) Find the number of arrangements of all the letters of the word ENGINEERING? (ii) How many different strings of length 4 can be formed?

2) In the word SOCIOLOGICAL, (i) How many arrangements are there for all the letters? (ii) In how many of these arrangements all vowels are adjacent?

3) Find The number of nonnegative integer solutions of the equation $X_1 + X_2 + X_3 + X_4 + X_5 = 8$

4) A certain question paper contains two parts A and B each containing 4 questions. How many different ways a student can answer 5 questions by selecting at least 2 questions from each part?

SIMP-Module-3

FUNCTIONS

1. Problems on COMPOSITION OF FUNCTIONS:

Examples:

(1) Let f,g, and h be functions from Z to Z defined by,

$f(x)=x-1$, $g(x)=3x$ and $h(x)=$

- Determine $(fo(goh))(x)$ and $((fog)oh)(x)$. (ii) Verify that $fo(goh)=(fog)oh$.

(2) Let f and g functions from R to R defined by

and $.If . Determine a and b values. (3) Let f,g,h be functions from R to R defined by $f(x)=x+2, g(x)=x-2$ and $h(x)=3x$. Find gof , fog , fof , hog and foh .$

2. Problems on No. Functions, one-to-one/ Injective and onto/ Surjective functions:

Examples:

(1) Let $A = \{1,2,3,4\}$ and $B = \{1, 2, 3, 4, 5, 6\}$ be two sets.

(i) Find how many functions are there from A to B ? How many of these are one to one? How many are onto?

(ii) Find how many functions are there from B to A ? How many of these are onto? How many are one to one?

(2) let $A=\{1,2,3,4,5,6,7\}$ and $B=\{w,x,y,z\}$. Find the number of onto functions from A to B .

(3) The functions $f:R \rightarrow R$ and $g:R \rightarrow R$ are defined by, $f(x)=3x+7$ and $g(x)=x(x^3-1)$. Verify that $f(x)$ is one-to-one but $g(x)$ is not.

(4) Consider the function $f:R \rightarrow R$ defined by $f(x)=x^2$. Determine whether f is one-to-one or onto. If f is not onto, find its range. Is f invertible?

3. Problems on INVERSE OF FUNCTION / INVERTIBLE FUNCTIONS:

Examples:

(1) Let $f: R \rightarrow R$ be defined by, $h(x)=$

Determine $f(0)$, $f(-1)$, $f^{-1}(1)$, $f^{-1}([-5, 5])$, $f^{-1}(3)$, $f^{-1}(-3)$, $f^{-1}(6)$, and $f^{-1}([-6, 5])$.

(2) If $f: A \rightarrow B$, $g: B \rightarrow C$ are invertible functions, then prove that $g \circ f: A \rightarrow C$ is an invertible function and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

(3) Let $A=B=R$ be the set of real numbers, the functions f and g from R to R defined by and . Show that, each of f and g is the inverse of the other. (4) Consider two functions $f(x)=2x+5$ and $g(x)=$ Prove that, g is an inverse of f .

SIMP-Module-4

(I) Problems on RECURRENCE RELATIONS:

Examples:

- (1) Find the recurrence relation and the initial conditions for the sequence 0, 2, 6, 12, 20, 30, 42, and hence find the general term of the sequence.
- (2) Solve the recurrence relation,
 - (i) $a_n + a_{n-1} - 6a_{n-2} = 0$ for $n \geq 2$ with $a_0 = -1$ and $a_1 = 8$.
 - (ii) $a_n = 2(a_{n-1} - a_{n-2})$ for $n \geq 2$ with $a_0 = 1$ and $a_1 = 2$.
- (3) A sequence $\{C_n\}$ is defined recursively by, $C_n = 3C_{n-1} - 2C_{n-2}$ for all $n \geq 3$ with $C_1 = 5$ and $C_2 = 3$ as the initial conditions, show that $C_n = -2^n + 7$.
- (4) Solve the recurrence relation

$$D_n = bD_{n-1} - b^2D_{n-2} \text{ for } n \geq 3, \text{ given that } D_1 = b > 0 \text{ and } D_2 = 0.$$
- (5) Solve the following Recurrence Relation

$$a_{n+2}^2 - 5a_{n+1}^2 + 4a_n^2 = 0 \text{ for } n \geq 0, \text{ given } a_0 = 4 \text{ and } a_1 = 13.$$
- (6) The number of virus affected files in a system is 1000 (to start with) and this increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. etc.,

(II) Problems on PRINCIPLE OF INCLUSION AND EXCLUSION:

Examples:

- (1) How many integers between 1 and 300 (inclusive) which are divisible by (i) at least one of 5, 6, 8 (ii) divisible by none of 5, 6, 8? (iii) at least two of 5, 6, 8. (i) exactly two of 5, 6, 8 and (ii) divisible by at least two of 5, 6, 8.
- (2) Determine the number of positive integers such that $1 \leq n \leq 100$ and is not divisible by 2, 3 or 5.
- (3) In a survey of 260 students, the following data were obtained. 64 had taken mathematics, 94 had taken CS, 58 had taken EC, 28 had taken both mathematics and EC, 26 had taken both mathematics and CS, 22 had taken both CS and EC, and 14 had taken all three types of courses. Determine how many of these students had taken none of the three courses.
- (4) In how many ways can the 26 letters of the English alphabet be permuted so that none of the patterns CAR, DOG, PUN or BYTE occurs?
- (5) In how many ways can one arrange the letters in the word CORRESPONDENTS so that there is no pair of consecutive identical letters? etc.,

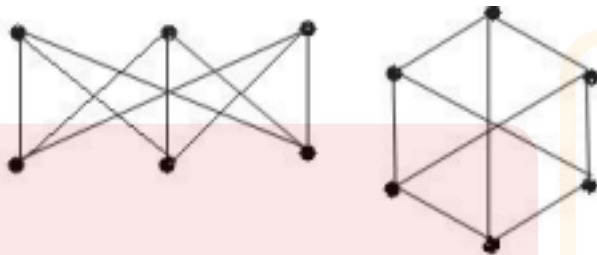
SIMP-Module-5

1. DEFINITIONS with example for each:

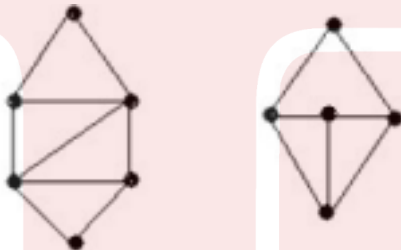
Graph, Simple Graph, Complement, Isomorphism, Multi-Graph, Complete Graph, Induced Subgraph, Regular Graph, Degree, Sub Graph, Path, Cycle, Circuit, Walk, Trail, Spanning Subgraph, Tree, Forest, Rooted Tree, Spanning Tree, Binary Tree, Complete Binary Tree, Full Binary Tree, Prefix Code, Balanced Binary Tree, Order of a Graph $|V|$, Connected Graph, Disconnected Graph, Pendant Vertex, Weight of a Tree, Optimal Tree Etc.,

2. Problems on TWO GRAPHS G AND H ARE ISOMORPHIC OR NOT :

Examples: (1) Define Isomorphism. Determine whether the following graphs are isomorphic.



(2) Verify the two graphs are isomorphic or not.



3. Problems on OPTIMAL PREFIX CODE: (Symbols with frequencies) Examples:

(1) Construct an optimal prefix code for the symbols a,b,c,d,e,f,g,h,i,j that occur with respective frequencies 78,16,30,35,125,31,20,50,80,3.

(2) Construct an optimal prefix code for the symbols a, o, q, u, y, z that occur with frequencies 20, 28, 4, 17, 12, 7 respectively.

4. Problems on OPTIMAL PREFIX CODE: (For a given message)

Examples:

- (1) Obtain the optimal prefix code for the message ROAD IS GOOD. Indicate the code.
- (2) Obtain an optimal prefix code for the message LETTER RECEIVED. Indicate the code.
- (3) Obtain the prefix code for the message MISSION SUCCESSFUL. Indicate the code.

5. Problems on OPTIMAL TREE (OR) HUFFMAN TREE:

Example: Define optimal tree and construct an optimal tree for a given set of weights $\{4,15,25,8,16\}$. Hence find the weight of the optimal tree.

Complete QB-Module-1

(I) DEFINITIONS with example for each:

Proposition, Tautology, Contradiction, Open sentence, Disjunction (OR), Conjunction (AND), Negation, Quantifier, compound statement, Converse, Inverse, and ContraPositive. etc.,

(II) Problems on TAUTOLOGY, CONTRADICTION, CONTINGENCY:

Examples:

(1) Prove that, for any propositions p, q, r, the compound propositions, are tautologies. i) $\{(\rightarrow) \wedge (\rightarrow) \} \rightarrow \{ (\rightarrow) \}$ ii) $\{ \rightarrow (\rightarrow) \} \rightarrow \{ (\rightarrow) \rightarrow (\rightarrow) \}$

(2) Determine whether the following statement is tautology or not.
 $(\rightarrow (\vee)) \leftrightarrow ((\wedge \neg) \rightarrow)$

(III) Problems on LOGICALLY EQUIVALENT STATEMENTS: (using truth tables)

Examples:

(1) By constructing the truth table, show that the compound propositions $\wedge (\neg \vee)$ and $\vee (\wedge \neg)$ are not logically equivalent.

(2) Use truth tables to verify,

(i) $[\rightarrow (\wedge) \leftrightarrow (\rightarrow) \wedge (\rightarrow)]$ (ii) $[(\vee) \rightarrow] \leftrightarrow [(\rightarrow) \wedge (\rightarrow)]$

(iii) $[(\leftrightarrow) \wedge (\leftrightarrow) \wedge (\leftrightarrow)] \leftrightarrow [(\rightarrow) \wedge (\rightarrow) \wedge (\rightarrow)]$

(IV) Problems on LOGICALLY EQUIVALENT STATEMENT USING Laws of Logic:

Examples:

(1) Define logical equivalence of two propositions. Prove the following logical equivalences without using the truth tables (using laws of logic):

i) $\vee [\wedge (\vee)] \leftrightarrow$ ii) $[(\neg \vee \neg) \rightarrow (\wedge \wedge) \leftrightarrow \wedge$

(iii) $(\rightarrow) \wedge (\neg \wedge (\vee \neg)) \leftrightarrow \neg [\vee]$

(2) Show that $[(\vee) \wedge \neg (\neg \wedge (\neg \vee \neg))] \vee (\neg \wedge \neg) \vee (\neg \wedge \neg)$ is tautology using laws of logic.

(V) Problems on TRUTH TABLES AND INDEPENDENTS OF ITS COMPONENTS:

Examples:

(1) Find the possible truth values of p, q and r if (i) $\rightarrow (\vee)$ is FALSE

(ii) $\wedge (\rightarrow)$ is TRUE

(2) Show that $(\wedge (\rightarrow)) \rightarrow$ is independent of its components.

- (3) Let p, q be primitive statements for which the implication $\rightarrow$ is false. Determine the truth values for each of the following: (i) $\wedge$ (ii) $\neg \vee$ (iii) $\rightarrow$
- (4) Let p, q, r be propositions having truth values 0, 0 and 1 respectively. Find the truth values of the following compound propositions: (i) $(\wedge) \rightarrow$ (ii) $\rightarrow (\wedge)$ (iii) $\wedge (\rightarrow)$ (iv) $\rightarrow (\rightarrow \neg)$.
- (5) Show that the truth values of the following statements are independent of their components: (i) $[\wedge (\rightarrow)] \rightarrow$ (ii) $(\rightarrow) \leftrightarrow [\neg \vee]$

(VI) Problems on DUAL AND PRINCIPLE OF DUALITY:

Examples:

- (1) Verify the principle of duality for the logical equivalence:
 $\sim(\wedge) \rightarrow \sim \vee (\sim \vee) \Leftrightarrow \sim \vee$.
- (2) Define a dual logical statement. Write the dual of the following logical statements: (i) $(\vee_0) \wedge (\vee) \vee (\wedge \wedge_0)$ (ii) $(\wedge) \vee_0$ (iii) $[\neg(\vee) \wedge \{ \vee \neg(\wedge \neg) \}]$

(VII) Problems on DIRECT PROOF, PROOF BY CONTRADICTION, INDIRECT PROOF:

Examples:

- (1) Give : (i) A direct proof (ii) An indirect proof and (iii) Proof by contradiction, for the following statement: "If n is an even integer, then $(n + 7)$ is an even integer".
- (2) Give a direct proof for each of the following. (i) For all integers a and b , if a and b are both even, then $a + b$ is even. (ii) For all integers a and b , if a and b are even, then ab is even.
- (3) Prove that for every integer n , n^2 is even if and only if n is even.
- (4) Give a direct proof of the statement "the square of an odd integer is an odd integer". (5) Give (i) direct proof (ii) indirect proof (iii) proof by contradiction for the following statement: "if n is an odd integer then $n+9$ is an even integer".

(VIII) Problems on QUANTIFIERS, TRUTH VALUES OF QUANTIFIED STATEMENTS:

- (1) Determine the truth value of each of the following quantified statements for the set of all non-zero integers: (i) $\exists, \exists [= 1]$ (ii) $\forall, \exists [= 1]$ (iii) $\exists, \exists [(2 + = 5) \wedge (-3 = -8)]$ (iv) $\exists, \exists, [(3 - = 17) \wedge (2 + 4 = 3)]$ (v) $\exists, \forall, [= 1]$
- (2) Determine the truth value of each of the following quantified statements for the set of all non-zero integers: (i) $\exists, \exists, [= 2]$ (ii) $\forall, \exists, [= 2]$ (iii) $\exists, \forall, [= 2]$ (iv) $\exists, \exists, [(3 + = 8) \wedge (2 - = 7)]$ (v) $\exists, \exists, [(4 + 2 = 3) \wedge (- = 1)]$
- (3) Consider the following open statements with the set of all real numbers as the universe, $(p): \geq 0, (q): \geq 0, q(p): \geq 0, p - 4 = 0$ and $(r): \geq 0, p - 3 > 0$, then find the truth values of (i) $\exists, [(p) \wedge (q)]$ (ii) $\forall, [(p) \rightarrow (q)]$ (iii) $\forall, [(p) \rightarrow (r)]$
- (4) Let $(p): \geq 0, q(p): \geq 0$ and $(r): \geq 0, p - 3 - 4 = 0$. Then for the universe comprising of all real

numbers, find the truth values of, (i) $\exists, [() \wedge ()]$ (ii) $\forall, [() \rightarrow ()]$ (iii) $\exists, [() \wedge ()]$ (iv) $\forall, [() \rightarrow ()]$ (v) $\forall, [() \rightarrow ()]$ (vi) $\forall, [() \vee ()]$

(IX) Problems on CONVERSE, INVERSE, CONTRAPOSITIVE PROBLEMS:

Examples:

- (1) Find converse and contrapositive of the statement: $p \rightarrow (q)$.
- (2) Consider the sentence, "if $5x-1=9$ then $x=2$ " i.e., $p \rightarrow q$. Find converse, inverse and contrapositive statements.

(X) Problems on VALIDITY OF THE ARGUMENT:

- (1) Prove that the following argument is valid.

$$\forall, [() \rightarrow ()]$$

$$\forall, [() \rightarrow ()]$$

$$\therefore \forall, [() \rightarrow ()]$$

- (2) Establish the validity of the following Argument

$$\rightarrow r$$

$$\sim \rightarrow q$$

$$q \rightarrow s$$

$$\therefore \sim r \rightarrow s$$

MODULE-2

(I) Problems on MATHEMATICS INDUCTION:

Examples:

- (1) By the principle of Mathematical Induction, prove that,

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

- (2) Define Principle of Mathematical Induction. For the Fibonacci sequence

Prove that,

- (3) By Mathematical Induction, Prove that, for any positive integer n ,

the number $= 5 + 2 \times 3^{n-1} + 1$ is a multiple of 8.

[OR] Prove by mathematical induction, for every integer 8 divides $5^n + 2 \times 3^n - 1 + 1$.

- (5) Prove that $4n < (n^2 - 7)$ for all positive integers $n \geq 6$

- (6) Prove by mathematical induction that, for every positive integer n , 5 divides $(n^5 - n)$

Problems on BINOMIAL THEOREM:

Examples: (1) Find the coefficient of $x^9 y^3$ in the expansion of $(2x-3y)^{12}$ (2) Find the coefficient of $x^9 y^3$ in the expansion of $(2x-3y)^{12}$ etc.,

(III) Problems on MULTINOMIAL THEOREM:

Example: (1) Find the coefficient of $a^2 b^3 c^2 d^5$ and $a^3 b^4 c^2 d^2$ in the expansion of $(a+2b-3c+2d+5)^{16}$

(2) Find the number of distinct terms in the expansion of $(x_1+x_2+x_3+x_4+x_5)^{16}$

(3) Find the coefficient of xyz^2 in the expansion of $(2x-y-z)^4$ etc.,

(IV) Problems on PERMUTATIONS, COMBINATIONS AND PRODUCT

RULE:

Examples:

- 1) In the word ENGINEERING, (i) Find the number of arrangements of all the letters of the word ENGINEERING? (ii) How many different strings of length 4 can be formed?
- 2) In the word SOCIOLOGICAL, (i) How many arrangements are there for all the letters? (ii) In how many of these arrangements all vowels are adjacent?
- 3) Find The number of nonnegative integer solutions of the equation $X_1+X_2+X_3+X_4+X_5=8$
- 4) A certain question paper contains two parts A and B each containing 4 questions. How many different ways a student can answer 5 questions by selecting at least 2 questions from each part?
- 5) A certain question paper contains three parts A,B,C with four questions in part A, five questions in part B and six questions in part C. It is required to answer seven questions selecting at least two questions from each part. In how many ways can a student select his seven questions for answering?
- 6) How many positive integers n can we form using the digits 3,4,4,5,5,6,7 if we want n to exceed 5,000,000? etc.,

MODULE-3

Relations,(all)

RELATIONS

(I) DEFINITIONS with example for each:

Relation, Types/properties of relations (reflexive, transitive, symmetric, anti symmetric,

irreflexive, asymmetric), partial order relation, equivalence relation, lattice, Hasse diagram/poset diagram etc.,

(II) Problems on types of relations/properties of relations:

Examples:

- (1) Let $A = \{1, 2, 3, 4\}$, $R = \{(1, 3), (1, 1), (3, 1), (1, 2), (3, 3), (4, 4)\}$ be the relation on A . Determine whether the relation R is reflexive, irreflexive, symmetric, antisymmetric or transitive.
- (2) Let $A = \{1, 2, 3, 4\}$, and let R be the relation defined by $= \{(\cdot, \cdot) \mid \in, \leq\}$. Determine whether R is reflexive, symmetric, antisymmetric or transitive.

(III) Problems on EQUIVALENCE RELATION and PARTIAL ORDER RELATION :

Examples:

Type-1 problems:

- (1) Define a relation R on $A \times A$ by $(x_1, y_1)R(x_2, y_2)$ iff $x_1 + y_1 = x_2 + y_2$ where $A = \{1, 2, 3, 4, 5\}$. Verify that R is an equivalence relation.
- (2) Define a relation R on the set $A = \{1, 2, 3\}$ as follows. $R = \{(1, 1), (2, 3), (2, 2), (3, 3), (3, 2)\}$. Verify that R is an Equivalence relation and partial order relation.

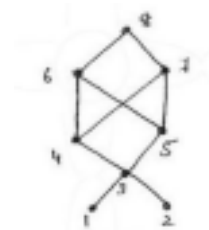
Type-2 problems:

Prove the following:

- (i) on the set of all integers, the relation R defined by aRb if and only if a is reflexive but not irreflexiv.
- (ii) On the set of all integers, the relation R defined by aRb if and only if $|a-b|=2$ is irreflexive and symmetric.

(IV) Problems on HASSE DIAGRAM/POSET DIAGRAM, LUB ,GLB etc.:

- (1) Draw Hasse diagram representing the positive divisors of 36 (i.e., D_{36}). (2) Let $A = \{1, 2, 3, 4, 6, 12\}$. On A define the relation R by aRb if and only if 'a' divides 'b'. Draw Hasse diagram and write down the matrix relation,
- (3) Consider the Hasse diagram of a Poset (A, R) as shown in figure. If $B = \{3, 4, 5\}$, find (i) all upper bounds of B (ii) all lower bounds of B (iii) the least upper bound of B (iv) the greatest lower bound of B (v) is this a Lattice (vi) maximal/greatest element of hasse diagram ?



- (4) Let $A = \{1, 2, 3, 6, 9, 8\}$ and R on A by xRy if $x|y$ (i.e., x divides y). Draw the Hasse

diagram for the post (A,R) .

(V) MISCELLANEOUS PROBLEMS ON RELATIONS:

Examples:

- (1) Matrix form/relation matrix of a given relation, Directed graph/digraph of a given relation, composition of relation $R \circ S$, R^2, R^3 etc., problems
- (2) Number of relations on a given set of problems, Equivalence classes etc., problems.

FUNCTIONS

(I) Problems on COMPOSITION OF FUNCTIONS:

Examples:

- (1) Let f, g , and h be functions from Z to Z defined by,
 $f(x)=x-1$, $g(x)=3x$ and $h(x)=$
 • Determine $(fo(goh))(x)$ and $((fog)oh)(x)$. (ii) Verify that $fo(goh)=(fog)oh$.
- (2) Let f and g functions from R to R defined by
 and .If . Determine a and b values. (3) Let f, g, h be functions from R to R defined by
 $f(x)=x+2, g(x)=x-2$ and $h(x)=3x$. Find gof , fog , fof , hog and foh .

(II) Problems on No. Functions, one-to-one/ Injective and onto/ Surjective functions:

Examples:

- (1) Let $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3, 4, 5, 6\}$ be two sets.
 (i) Find how many functions are there from A to B ? How many of these are one to one? How many are onto?
 (ii) Find how many functions are there from B to A ? How many of these are onto? How many are one to one?
- (2) let $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $B = \{w, x, y, z\}$. Find the number of onto functions from A to B .
- (3) The functions $f: R \rightarrow R$ and $g: R \rightarrow R$ are defined by, $f(x)=3x+7$ and $g(x)=x(x^3-1)$.
 Verify that $f(x)$ is one-to-one but $g(x)$ is not.
- (4) Consider the function $f: R \rightarrow R$ defined by $f(x)=x^2$. Determine whether f is one-to-one or onto. If f is not onto, find its range. Is f invertible?

(III) Problems on INVERSE OF FUNCTION / INVERTIBLE FUNCTIONS:

Examples:

(1) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by, $h(x)=$

Determine $f(0)$, $f(-1)$, $f^{-1}(1)$, $f^{-1}([-5, 5])$, $f^{-1}(3)$, $f^{-1}(-3)$, $f^{-1}(6)$, and $f^{-1}([-6, 5])$.

(2) If $f: A \rightarrow B$, $g: B \rightarrow C$ are invertible functions, then prove that $g \circ f: A \rightarrow C$ is an invertible function and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

(3) Let $A=B=\mathbb{R}$ be the set of real numbers, the functions f and g from $\mathbb{R}$ to $\mathbb{R}$ defined by and . Show that, each of f and g is the inverse of the other. (4) Consider two functions $f(x)=2x+5$ and $g(x)=$ Prove that, g is an inverse of f .

MODULE-4

(I) Problems on RECURRENCE RELATIONS:

Examples:

(1) Find the recurrence relation and the initial conditions for the sequence 0,2,6,12,20,30,42,.....and hence find the general term of the sequence.

(2)
for with
for with

(3) A sequence $\{C_n\}$ is defined recursively by, for all with as the initial conditions, show that .

(4) Solve the recurrence relation

for , given that $D_1=b>0$ and $D_2=0$.

(5) Solve the following Recurrence Relation

for , given

(6) The number of virus affected files in a system is 1000 (to start with) and this increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. **etc.,**

(II) Problems on PRINCIPLE OF INCLUSION AND EXCLUSION:

Examples:

(1) How many integers between 1 and 300 (inclusive) which are divisible by (i) at least one of 5, 6, 8 (ii) divisible by none of 5, 6, 8? (iii) at least two of 5,6,8. (i) exactly two of 5,6,8 and (ii) divisible by at least two of 5,6,8.

- (2) Determine the number of positive integers such that $1 \leq n \leq 100$ and n is not divisible by 2, 3 or 5.
- (3) In a survey of 260 students, the following data were obtained. 64 had taken mathematics, 94 had taken CS, 58 had taken EC, 28 had taken both mathematics and EC, 26 had taken both mathematics and CS, 22 had taken both CS and EC, and 14 had taken all three types of courses. Determine how many of these students had taken none of the three courses.
- (4) In how many ways can the 26 letters of the English alphabet be permuted so that none of the patterns CAR, DOG, PUN or BYTE occurs?
- (5) In how many ways can one arrange the letters in the word CORRESPONDENTS so that there is no pair of consecutive identical letters? **etc.,**

(III) Problems on ROOK POLYNOMIAL:

Examples:

- (1) Define Rook Polynomial. Find the ROOK polynomial for 3X3 board using the expansion formula.
- (2) An apple, a banana, a mango and an orange are to be distributed among 4 boys B1, B2, B3 and B4. The boys B1 and B2 do not wish to have an apple, the boy B3 does not want a banana or mango and B4 refuses oranges. In how many ways the distribution can be made so that no boy is displeased.
- (3) Five teachers T1, T2, T3, T4, T5 are to be made class teachers for five classes C1, C2, C3, C4, C5, one teacher for each class. T1 and T2 do not wish to become the class teachers for C1 or C2, T3 and T4 for C4 or C5 and T5 for C3 or C4 or C5. (i) Construct Board for the given constraints. (ii) Find Rook polynomials (iii) In how many ways can the teacher be assigned work without displeasing any teacher.
- (4) Write expansion and product formula. Find the Rook polynomial for 2X2 board using the expansion formula.

(IV) Problems on DERANGEMENTS:

- Examples:** (1) Define derangements. Evaluate
- (2) Find the number of derangements of 1, 2, 3, 4. Also list out all derangements.
- (3) How many permutations of 1, 2, 3, 4, 5, 6, 7 are not derangements?

MODULE-5

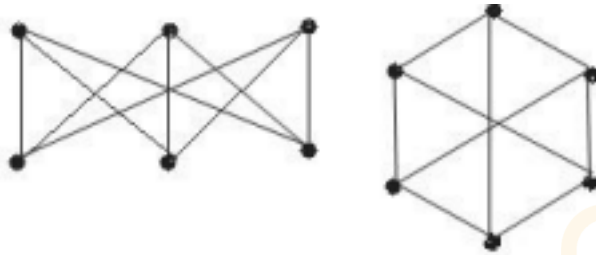
1) DEFINITIONS with example for each:

Graph, Simple Graph, Complement, Isomorphism, Multi-Graph, Complete Graph, Induced Subgraph, Regular Graph, Degree, Sub Graph, Path, Cycle, Circuit, Walk, Trail, Spanning Subgraph, Tree, Forest, Rooted Tree, Spanning Tree, Binary Tree, Complete Binary Tree, Full Binary Tree, Prefix Code, Balanced Binary Tree, Order of a Graph |V|,

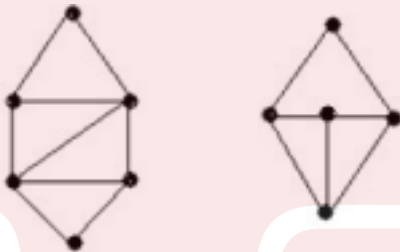
Connected Graph, Disconnected Graph, Pendant Vertex, Weight of a f A Tree, Optimal Tree Etc.,

II) Problems on TWO GRAPHS G AND H ARE ISOMORPHIC OR NOT :

Examples: (1) Define Isomorphism. Determine whether the following graphs are isomorphic.



(2) Verify the two graphs are isomorphic or not.



III) Problems on OPTIMAL PREFIX CODE: (Symbols with frequencies) Examples:

(1) Construct an optimal prefix code for the symbols a,b,c,d,e,f,g,h,i,j that occur with respective frequencies 78,16,30,35,125,31,20,50,80,3.

(2) Construct an optimal prefix code for the symbols a, o, q, u, y, z that occur with frequencies 20, 28, 4, 17, 12, 7 respectively.

IV) Problems on OPTIMAL PREFIX CODE: (For a given message)

Examples:

- (1) Obtain the optimal prefix code for the message ROAD IS GOOD. Indicate the code.
- (2) Obtain an optimal prefix code for the message LETTER RECEIVED. Indicate the code.
- (3) Obtain the prefix code for the message MISSION SUCCESSFUL. Indicate the code.

V) Problems on OPTIMAL TREE (OR) HUFFMAN TREE:

Example: Define optimal tree and construct an optimal tree for a given set of weights {4,15,25,8,16}. Hence find the weight of the optimal tree.

VI) PROBLEMS ON SORTING (MERGE SORT):

Example:

- (1) Apply the merge sort to the following given list of elements.
{-1,0,2,-2,3,6,-3,5,1,4}
- (2) Apply the merge sort the following list. Draw the splitting

and merging trees for each application of the procedure. merge sort to the list
 $\{6, 2, 7, 3, 4, 9, 5, 1, 8\}$

VII) Problems on DEGREE OF VERTEX and HANDSHAKING

THEOREM: Example:

(1) Determine the order $|V|$ of the graph $G=(V,E)$ in the following cases.

- (i) G is a regular graph with 15 edges . (ii) G has 10 edges with 2 vertices of degree 4 and all others of degree 3. (iii) G has 20 edges with 4 vertices of degree 3, 2 vertices of degree 4 and remaining vertices of degree (iv) G is a cubic graph with 9 edges.

(2) A tree has four vertices of degree 2, one vertex of degree 3, two vertices of degree 4, and one vertex of degree 5, how many pendant vertices (degree one) does it have? (3) If a tree T has four vertices of degree 2, one vertex of degree 3, two vertices of degree 4, and one vertex of degree 5, find the number of leaves (degree one) in T ?

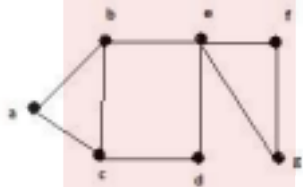
(4) Show that there exist no simple graphs corresponding to the following degree sequences:

- i) 0, 2, 2, 3, 4 ii) 1, 1, 2, 3 iii) 2, 3, 3, 4, 5, 6 iv) 2, 2, 4, 6

VIII) Problems on CYCLE, CIRCUIT, WALK, PATH, LENGTH OF A CYCLE:

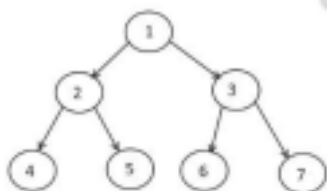
Example: In the graph shown below, determine the following:

- i) Walk from b to d that is not a trail. ii) $b - d$ trail that is not a path iii) path from b to d iv) closed walk from b to b that is not a circuit v) a circuit from b to b that is not a cycle vi) a cycle from b to b . vii) Cycle from e to e with length 5.



IX) Problems on PREORDER, POSTORDER AND INORDER TRAVERSAL:

Example: For the tree shown below, list the vertices according to a *preorder*, *inorder* and *postorder* traversal.



(X) Important THEOREMS:

- (1) Show that a tree with n vertices has an $n-1$ edge. [or] Show that, in a tree $|E|=|V|-1$
 [or] Prove that, in a tree, $|V|=|E|+1$
 (2) Prove that in a graph, the number of vertices of odd degree is even.

(3) State and prove Handshaking theorem. [or] Prove that in a graph, the sum of the degrees of all the vertices is an even number.

